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Introduction

The Mathematics Curriculum Guide serves as a guide for teachers when planning instruction and assessment. It defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessment. It provides additional guidance to teachers as they develop an instructional program appropriate for their students. It also assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This Guide delineates in greater specificity the content that all teachers should teach and all students should learn.

The format of the Curriculum Guide facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each objective. The Curriculum Guide is divided into sections: Curriculum Information, Essential Knowledge and Skills, Key Vocabulary, Essential Questions and Understandings, Teacher Notes and Elaborations, Resources, and Sample Instructional Strategies and Activities. The purpose of each section is explained below.

Curriculum Information:
This section includes the objective, focus or topic, and in some, not all, foundational objectives that are being built upon.

Essential Knowledge and Skills:
Each objective is expanded in this section. What each student should know and be able to do in each objective is outlined. This is not meant to be an exhaustive list nor is a list that limits what taught in the classroom. This section is helpful to teachers when planning classroom assessments as it is a guide to the knowledge and skills that define the objective.

Key Vocabulary:
This section includes vocabulary that is key to the objective and many times the first introduction for the student to new concepts and skills.

Essential Questions and Understandings:
This section delineates the key concepts, ideas, and mathematical relationships that all students should grasp to demonstrate an understanding of the objectives.

Teacher Notes and Elaborations:
This section includes background information for the teacher. It contains content that is necessary for teaching this objective and may extend the teachers’ knowledge of the objective beyond the current grade level. It may also contain definitions of key vocabulary to help facilitate student learning.

Resources:
This section lists various resources that teachers may use when planning instruction. Teachers are not limited to only these resources.

Sample Instructional Strategies and Activities:
This section lists ideas and suggestions that teachers may use when planning instruction.
The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:
- Identify the domain and range for a relation, given a set of ordered pairs, a table, or a graph.
- For each $x$ in the domain of $f(x)$, find $f(x)$.
- Identify the zeros of the function algebraically and confirm them, using the graphing calculator.
- Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically.
- Recognize restricted/discontinuous domains and ranges.
- Recognize graphs of parent functions for linear, quadratic, exponential, and logarithmic functions.
- Identify $x$-intercepts (zeros), $y$-intercepts, symmetry, asymptotes, intervals for which the function is increasing/decreasing, end behaviors; and asymptotes.
- Describe continuity of a function on its domain or at a point.
- Express intervals using correct interval notation and/or a compound inequality.
- Compare and contrast linear, quadratic, exponential, and logarithmic functions.

Key Vocabulary
- asymptote
- continuous functions
- discontinuous functions
- domain
- end behavior
- family of functions
- function
- function notation

Essential Questions
- What is a function?
- What is average rate of change and how can it be determined?
- When data is given in the form of a graph, table, rule, or words, how is it recognized as a linear, quadratic, exponential, or logarithmic function?
- How does a function representing a set of data correlate to the parent linear function?
- What is the best representation of a data set (table, graph, equation) given a real-world situation?
- What is the zero of a function?
- What is the relationship between domain and range?
- In a real-world problem, what is the difference between the domain of a function and the practical domain of the model?
- In a real-world problem, what is the difference between the range of a function and the practical range of the model?
- How are the $x$- and $y$-intercepts determined, and what is their significance in real-world situations?
- In a real-world problem, how can the continuity of a function be determined?
- How does the leading coefficient determine the end behavior of a linear, quadratic, exponential, or logarithmic function?
- How can the graphing calculator be used to investigate linear, quadratic, exponential, or logarithmic functions?

Essential Understandings
- The domain of a function consists of the first coordinates of the ordered pairs that are elements of a function. Each element in the domain is an input into the independent variable of the function.
- The range of a function consists of the second coordinates of the ordered pairs that are elements of a function. Each element in the range is an output in the dependent variable of a function.
- For each $x$ in the domain of $f$, $x$ is a member of the input of the function $f$ and $f(x)$ is a member of the output of $f$, and the ordered pair $[x, f(x)]$ is a member of $f$.
- A value $x$ in the domain of $f$ is an $x$-intercept or a zero of a function $f$ if and only if $f(x) = 0$.
- Functions describe the relationship between two variables where each input is paired to a unique output.
- Functions are used to model real-world phenomena.
- A function is increasing on an interval if its graph, as read from left to right, is rising in that interval.
- A function is decreasing on an interval if its graph, as read from left to right, is going down in that interval.
- Exponential and logarithmic functions are either strictly increasing or strictly decreasing.
- A function is continuous on an interval if the function is defined for every value in the interval and there are no breaks in the graph. A continuous function can be drawn without lifting the pencil.
- A turning point is a point on a continuous interval where the graph changes from increasing to decreasing or from decreasing to increasing.
- A function, $f$, has a local maximum in some interval at $x = a$, if $f(a)$ is the largest value of $f$ in that interval.
**Topic**
Algebra and Functions

**Virginia SOL AFDA.1**
The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include:

a. continuity;
b. local and absolute maxima and minima;
c. domain and range;
d. zeros;
e. intercepts;
f. intervals in which the function is increasing/decreasing;
g. end behaviors; and
h. asymptotes.

**Key Vocabulary (continued)**
- local maximum
- local minimum
- parent function
- practical domain
- practical range
- range
- turning point (vertex)
- x-intercept (horizontal intercept)
- y-intercept (vertical intercept)
- zeros of the function
- exponential function
- logarithmic function
- linear function
- quadratic function

**Essential Understandings (continued)**
- A function, $f$, has a *local minimum* in some interval at $x = a$, if $f(a)$ is the smallest value of $f$ in that interval.
- Asymptotes can be used to describe local behavior and end behavior of graphs. They are lines or other curves that approximate the graphical behavior of a function.
- The following statements are equivalent:
  - $k$ is a zero of the polynomial function $f$;
  - $k$ is a solution of the polynomial equation $f(x) = 0$;
  - $k$ is an $x$-intercept for the graph of the polynomial; and
  - $(x - k)$ is a factor of the polynomial.
- Continuous and discontinuous functions can be identified by their equations or graphs. The *end behavior* of a function refers to the graphical behavior of a function as $x$ goes to positive and negative infinity.

**Teacher Notes and Elaborations**
A set of data may be characterized by patterns and those patterns can be represented in multiple ways. Algebra is a tool for describing patterns, generalizing, and representing a relationship in which output is related to input. Mathematical relationships are readily seen in the translation of quantitative patterns and relations to equations or graphs. Collected data can be organized in a table or visualized in a graph and analyzed for patterns.

In a function, the relationship between the domain and range may be represented by a rule. This rule may be expressed using *function notation*, $f(x)$, which means the value of the function at $x$ and is read "$f$ of $x$".

A *function* is a special relation in which each different input value is paired with exactly one output value (a unique output for each input). The set of input values forms the *domain* and the set of output values forms the *range* of the function. Sets of ordered pairs that do not represent a function should also be identified.

The collection of all possible values of the independent variable is called the domain of the function. The *practical domain* is the collection of replacement values of the independent variable that makes practical sense in the context of the problem. For example: given $c = fg$ where $c$ is the cost of filling a gas tank in a car and $g$ is the number of gallons, the practical domain are the values of $g$ that make sense (0 to the maximum capacity of the gas tank). The collection of all possible values of the dependent variable is the range of the function. The *practical range* corresponds to the practical domain.

For the exponential parent function, the domain is all real numbers and the range is all positive real numbers. For the logarithmic parent function, the domain is all positive real numbers and the range is all real numbers.

(continued)
**Teacher Notes and Elaborations (continued)**

The graphs/equations for a family of functions can be determined using a transformational approach. A *family of functions* is a group of functions with common characteristics. A *parent function* is the simplest function with these characteristics. A parent function and one or more transformations make up a family of functions.

<table>
<thead>
<tr>
<th>Parent Function</th>
<th>Family of Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = x$</td>
<td>$f(x) = x + 1$</td>
</tr>
<tr>
<td>$f(x) = 3x$</td>
<td>$f(x) = -\frac{1}{2}x$</td>
</tr>
</tbody>
</table>

Graphs can be used as visual representations to investigate relationships between quantitative data. Graphically, a function may be determined by applying the vertical line test (A graph is a function if there exists no vertical line that intersects the graph in more than one point).

**Zeros of the function** (roots or solutions) are the $x$-intercepts of the function and are found algebraically by substituting 0 for $y$ and solving the subsequent equation. An object $x$ in the domain of $f$ is an $x$-intercept or a zero of a function $f$ if and only if $f(x) = 0$.

If the average rate of change in $x$ with respect to $y$ remains constant for any two points in a data set, the points will lie on a straight line. That is, $y$ is a linear function of $x$. The graph of a *linear function* is a line.

A quadratic equation is a polynomial equation containing a variable to the second degree. The standard form of a quadratic equation is $y = ax^2 + bx + c$, where $a$, $b$, and $c$ are real numbers and $a \neq 0$. The graph of a *quadratic function* is called a parabola. Quadratic functions can be written in the form $y = a(x - h)^2 + k$ to facilitate graphing by transformations and finding maxima and minima. The chart below summarizes the characteristics of the graph of $y = a(x - h)^2 + k$.

<table>
<thead>
<tr>
<th>$y = a(x - h)^2 + k$</th>
<th>$a$ is positive</th>
<th>$a$ is negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex</td>
<td>$(h, k)$</td>
<td>$(h, k)$</td>
</tr>
<tr>
<td>Axis of Symmetry</td>
<td>$x = h$</td>
<td>$x = h$</td>
</tr>
<tr>
<td>Direction of Opening</td>
<td>upward (minima)</td>
<td>downward (maxima)</td>
</tr>
</tbody>
</table>

As the value of the absolute value of $a$ increases, the graph of $y = a(x - h)^2 + k$ narrows.

An *exponential function* is a function of the form $y = a^x$, where $a$ is a positive constant not equal to one. The inverse of this is the *logarithmic function* $y = \log_a x$. Population growth and viral growth are among examples of exponential functions. Exponential and logarithmic functions are either strictly increasing or strictly decreasing. Exponential and logarithmic functions also have asymptotes.

An *asymptote* of a curve is a line such that the distance between the curve and the line approaches zero as they tend to infinity.

A function is *continuous* if the graph can be drawn without lifting the pencil from the paper. A graph is *discontinuous* if it has jumps, breaks, or holes in it.

Graphing calculators are a tool for investigating the shape and behavior of linear, quadratic, exponential, and logarithmic functions.
Curriculum Information

**Topic**
Algebra and Functions

**Virginia SOL AFDA.2**
The student will use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).

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<table>
<thead>
<tr>
<th>Essential Knowledge and Skills</th>
<th>Key Vocabulary</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:</td>
<td>exponential function</td>
</tr>
<tr>
<td>- Write an equation of a line when given the graph of a line.</td>
<td>linear function</td>
</tr>
<tr>
<td>- Recognize graphs of parent functions for linear, quadratic, exponential, and logarithmic functions.</td>
<td>logarithmic function</td>
</tr>
<tr>
<td>- Write the equation of a linear, quadratic, exponential, or logarithmic function in ((h, k)) form given the graph of the parent function and transformation information.</td>
<td>parabola</td>
</tr>
<tr>
<td>- Describe the transformation from the parent function given the equation written in ((h, k)) form or the graph of the function.</td>
<td>parent function</td>
</tr>
<tr>
<td>- Given the equation of a function, recognize the parent function and transformation to graph the given function.</td>
<td>quadratic function</td>
</tr>
<tr>
<td>- Recognize the vertex of a parabola given a quadratic equation in ((h, k)) form or graphed.</td>
<td>transformation</td>
</tr>
<tr>
<td>- Describe the parent function represented by a scatter plot.</td>
<td></td>
</tr>
</tbody>
</table>

**Key Vocabulary**
- exponential function
- linear function
- logarithmic function
- parabola
- parent function
- quadratic function
- transformation

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**Essential Questions**
- What is the change in the graph and the equation of a basic function as the graph is translated, reflected, or shows a vertical stretch or shrink?
- How is a transformation described?
- How is the parent function of a graph described?
- How is the type of graph determined when given the equation in \((h, k)\) form?
- How is the \(y=mx+b\) form of a linear equation derived when given in \((h, k)\) form?
- How is the \(y = ax^2 + bx + c\) form of a quadratic equation derived when given in \((h, k)\) form?
- How can the graphing calculator be best used to demonstrate transformations?

**Essential Understandings**
- Knowledge of transformational graphing using parent functions can be used to generate a mathematical model from a scatter plot that approximates the data.
- **Transformations** include:
  - Translations (horizontal and vertical shifting of a graph)
  - Reflections
  - Dilations (stretching and compressing graphs and)
  - Rotations
- The equation of a line can be determined by two points on the line or by the slope and a point on the line.

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**Teacher Notes and Elaborations**
A graph is a picture of an equation. The graphs of linear equations are straight lines.

The slope of a line can be described as \(\frac{\text{rise}}{\text{run}}\) or \(\frac{\text{change in } y}{\text{change in } x}\).

The slope \(m\) of a line that passes through the points \((x_1, y_1)\) and \((x_2, y_2)\) is \(m = \frac{y_2 - y_1}{x_2 - x_1}\). Students should understand that the subscripts are not interchangeable with exponents.

Linear equations can be written in a variety of forms:
- **Slope-intercept**: \(y = mx + b\), where \(m\) is the slope and \(b\) is the \(y\)-intercept.
- **Standard**: \(Ax + By = C\), where \(A\), \(B\), and \(C\) are integers and \(A\) is positive.
- **Point-slope**: \(y - y_1 = m(x - x_1)\)
- **Vertical line**: \(x = a\)
- **Horizontal line** (constant function): \(y = b\)

An appropriate technique for graphing a linear equation is determined by the given information and/or the tools available. Appropriate techniques for graphing a linear equation are: slope and \(y\)-intercept, \(x\)- and \(y\)-intercepts, and graphing by transformations.

(continued)
### Teacher Notes and Elaborations (continued)

A line can be represented by its graph or by an equation. The equation of a line defines the relationship between two variables. The graph of a line represents the set of points that satisfies the equation of a line. Equations of the line may be written using two ordered pairs (two points on the line), the $x$- and $y$-intercepts, or the slope and a point on the line.

If given two ordered pairs, the slope may be found by dividing the change in the $y$-coordinates by the change in the $x$-coordinates. Then, using the slope and one of the coordinates, the equation may be written using the point-slope form of the equation, $y - y_1 = m(x - x_1)$.

If the $x$- and $y$-intercepts are given, using these two points the equation of the line may be determined just as it is with any two points on the line.

The graph of a linear function is a line. The graph of a quadratic function is a parabola (U-shaped curve). The graph of an exponential or logarithmic function is a curve that shows increasing growth or decay.

*Parent functions* for linear, quadratic, exponential and logarithmic functions take the following forms:

- linear $y = x$
- quadratic $y = x^2$
- exponential $y = ab^x$, $a \neq 0$, $b > 0$, $b \neq 1$
- logarithmic $y = \log_b x$

To describe the transformation from the parent function for $y = x - k$, shift the parent function $k$ units down. The parent function is $y = x$, and the transformation is a translation of $-k$.

Given a quadratic equation in the form $y = a(x - h)^2 + k$, the vertex is $(h, k)$ and if $a > 0$, the parabola opens upward.

Quadratic equations can be written in $(h, k)$ form ($y = a(x - h)^2 + k$). Characteristics about the graph including whether the parabola opens up or down, whether it is wide or narrow, and the location of the vertex can be determined by looking at the values for $a$, $h$, and $k$.

A quadratic equation is a polynomial equation containing a variable to the second degree. The standard form of a quadratic equation is $y = ax^2 + bx + c$, where $a$, $b$, and $c$ are real numbers and $a \neq 0$. The graph of a quadratic function is called a parabola. Quadratic functions can be written in the form $y = a(x - h)^2 + k$ to facilitate graphing by transformations and finding maxima and minima. The chart below summarizes the characteristics of the graph of $y = a(x - h)^2 + k$.

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As the value of the absolute value of $a$ increases, the graph of $y = a(x - h)^2 + k$ narrows.

The use of scatterplots on a graphing calculator will determine if the relationship is polynomial, exponential, or logarithmic.

Graphing calculators can be used to determine the model of best fit for data presented in a scatter plot.
<table>
<thead>
<tr>
<th>Curriculum Information</th>
<th>Essential Knowledge and Skills</th>
<th>Essential Questions and Understandings</th>
<th>Teacher Notes and Elaborations</th>
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</thead>
<tbody>
<tr>
<td><strong>Topic</strong></td>
<td>Algebra and Functions</td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:</td>
<td>Essential Questions</td>
</tr>
<tr>
<td><strong>Virginia SOL AFDA.3</strong></td>
<td>The student will collect data and generate an equation for the curve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world problems or applications. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.</td>
<td>• Write an equation for the line of best fit, given a set of data points in a table, on a graph, or from a practical situation. • Make predictions about unknown outcomes, using the equation of a line of best fit. • Collect and analyze data to make decisions and justify conclusions. • Investigate scatter plots to determine if patterns exist, and identify the patterns. • Find an equation for the curve of best fit for data, using a graphing calculator. Models will include linear, quadratic, exponential, and logarithmic functions. • Make predictions, using data, scatter plots, or equation of curve of best fit. • Given a set of data, determine the model that would best describe the data. • Describe the errors inherent in extrapolation beyond the range of the data. • Estimate the correlation coefficient when given data and/or scatter plots.</td>
<td>• What is a curve of best fit? • How can the equation for the curve of best fit be found? • How is a curve of best fit used to make predictions in real-world situations? • Given a set of data points, how is a scatterplot constructed? • Given a scatterplot, how is the positive or negative relationship between the two variables described? • What determines a strong correlation and is there a direct relationship between correlation and cause and effect? • How can the graphing calculator be best used to plot the scatterplot and determine the line of best fit for all equations? • What does the line of regression show or tell in a linear equation?</td>
</tr>
<tr>
<td><strong>Key Vocabulary</strong></td>
<td>correlation coefficient curve of best fit least squares line residuals trend line</td>
<td>Essential Understandings</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• The regression equation modeling a set of data points can be used to make predictions where appropriate. • Data and scatter plots may indicate patterns that can be modeled with a function. • Graphing calculators can be used to collect, organize, picture, and create an algebraic model of the data. • Data that fit linear, quadratic, exponential, and logarithmic models arise from practical situations. • Two variables may be strongly associated without a cause-and-effect relationship existing between them. • Each data point may be considered to be comprised of two parts: fit (the part explained by the model) and residual (the result of chance variation or of variables not measured). • Residual = Actual – Fitted • Least squares regression generates the equation of the line that minimizes the sum of the squared distances between the data points and the line. • A correlation coefficient measures the degree of association between two variables that are related linearly.</td>
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<tr>
<td></td>
<td></td>
<td>Teacher Notes and Elaborations</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>When real-life data is collected, the data graphed usually does not form a perfectly straight line or a perfect quadratic curve. However, the graph may approximate a linear or quadratic relationship. When this is the case, a curve of best fit can be drawn, and a prediction equation that models the data can be determined. A curve of best fit for the relationship among a set of data points can be used to make predictions where appropriate. Accuracy of the equation can depend on sample size, randomness, and bias of the collection.</td>
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<td>A linear curve of best fit (line of best fit) may be determined by drawing a line connecting any two data points that seem to best represent the data. An equal number of points should be located above and below the line. This line represents the equation to be used to make predictions.</td>
<td>(continued)</td>
</tr>
</tbody>
</table>
**Curriculum Information** | **Essential Questions and Understandings**
---|---
**Teacher Notes and Elaborations** (continued)  
A quadratic curve of best fit may be determined by drawing a graph connecting any three points that seem to best represent the data. Putting these data points into a graphing calculator will result in a quadratic function. Since different people may make different judgments for which points should be used, one person’s equation may differ from another’s.

The graphing calculator can be used to determine the equation of the curve of best fit for linear, quadratic, exponential, and logarithmic.

A trend line is a line that approximates the relationship between the data sets of a scatter plot. It can be used to make predictions.

If a logical relationship exists between two variables, a graph is used to plot the available data. A scatterplot contains an x (independent or explanatory) value and a y (dependent or response) value.

A scatterplot serves two purposes:
1. it helps to see if there is a useful relationship between the two variables; and
2. it helps to determine the type of equation to use to describe the relationship.

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**Virginia SOL AFDA.3**
The student will collect data and generate an equation for the curve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world problems or applications. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.

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A trend line is a line that approximates the relationship between the data sets of a scatter plot. It can be used to make predictions.
<table>
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<tbody>
<tr>
<td><strong>Topic</strong></td>
<td><strong>Teacher Notes and Elaborations (continued)</strong></td>
</tr>
<tr>
<td>Algebra and Functions</td>
<td>In order to measure how closely points tend to cluster about the least-squares line, statisticians use a correlation coefficient, denoted $r$. If the data fit perfectly on a line with positive slope, the correlation coefficient is said to be +1. If the data fit perfectly on a line with negative slope, the correlation coefficient is said to be -1. If the points tend not to lie on any line, then the correlation coefficient is close to 0. Some calculators will compute the correlation coefficient.</td>
</tr>
<tr>
<td><strong>Virginia SOL AFDA.3</strong></td>
<td>Associated with every point is its vertical distance to the line. The sum of all the distances is a measure of how well the line fits the points. The smaller the sum of the distances, the better the fit. To find the line that best fits the data, statisticians prefer to work with the sum of the squares of these distances, rather than the vertical distances themselves. The line that minimizes the sum of these squares is called the least-squares line.</td>
</tr>
<tr>
<td></td>
<td>The correlation coefficient for a set of data $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ is given by:</td>
</tr>
<tr>
<td></td>
<td>$r = \frac{\bar{x} \bar{y} - \bar{x} \cdot \bar{y}}{s_x s_y}$</td>
</tr>
<tr>
<td></td>
<td>$= \frac{(\text{mean of the products}) - (\text{product of the means})}{(\text{product of the standard deviations})}$</td>
</tr>
<tr>
<td></td>
<td>More than one model for a set of data can be used. To determine which is a better model, analyze the differences between the $y$-values of the data and the $y$-values of each model. These differences are called residuals. The better model will have residuals that are closer to zero.</td>
</tr>
</tbody>
</table>
Curriculum Information  | Essential Knowledge and Skills  | Essential Questions and Understandings  | Teacher Notes and Elaborations
--- | --- | --- | ---
**Topic**  | **Key Vocabulary**  | **Essential Questions**  | **Teacher Notes and Elaborations**
Algebra and Functions  | The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:  
- Given an equation, graph a linear, quadratic, exponential, or logarithmic function with the aid of a graphing calculator.  
- Make predictions given a table of values, a graph, or an algebraic formula.  
- Describe relationships between data represented in a table, in a scatter plot, and as elements of a function.  
- Determine the appropriate representation of data derived from real-world situations.  
- Analyze and interpret the data in context of the real-world situation.  | - What are the ways in which data can be represented?  
- What is the best representation for a data set?  
- What is the relationship between the different forms of data representation?  | A set of data may be characterized by patterns and those patterns can be represented in multiple ways. Algebra is a tool for describing patterns, generalizing, and representing a relationship in which output is related to input. Mathematical relationships are readily seen in the translation of quantitative patterns and relations to equations or graphs. Collected data can be organized in a table or visualized in a graph and analyzed for patterns.

Pattern recognition and analysis might include:  
- patterns involving a given sequence of numbers;  
- a set of ordered pairs (i.e. relation) from a given pattern;  
- a pattern using the variable(s) in an algebraic format so that a specific term can be determined;  
- patterns demonstrated geometrically on the coordinate plane when appropriate; and  
- patterns that include the use of the ellipsis such as 1, 2, 3 . . . 99, 100.  

Patterns may be represented as relations and/or functions. A relation can be represented by a set of ordered pairs of numbers or paired data values. In an ordered pair, the first number is termed the abscissa (x-coordinate) and the second number is the ordinate (y-coordinate).  

A function is a special relation in which each different input value is paired with exactly one output value (a unique output for each input). The set of input values forms the domain and the set of output values forms the range of the function. Sets of ordered pairs that do not represent a function should also be identified. Graphs of functions with similar features are called a family of functions.  

Graphs can be used as visual representations to investigate relationships between quantitative data. Graphically, a function may be determined by applying the vertical line test (A graph is a function if there exists no vertical line that intersects the graph in more than one point).  

If a sequence does not have a last term, it is called an infinite sequence. Three dots called an ellipsis are used to indicate an omission. If a sequence stops at a particular term, it is a finite sequence.  

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<td>The student will transfer between and</td>
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<td>analyze multiple representations of</td>
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<td>**Teacher Notes and</td>
<td>Set builder notation is a method for</td>
<td>The set of all x such that x is an</td>
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<tr>
<td>Elaborations**</td>
<td>identifying a set of values. For</td>
<td>element of the real numbers.” The range</td>
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<td>example, the domain for y = x^2 – 5</td>
<td>for this equation would be written as y</td>
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</table>
|                        | would be written as \{x : x ∈ ℝ\}. This is read, “The set of all x such that x is an element of the real numbers.” The range for this equation would be written as \{y : y ≥ −5\}.
|                        |                                        | y ≥ −5}.
|                        |                                        |                                           |
|                        | To find the y-intercept in a quadratic |                                           |
|                        | function, let x = 0.                   |                                           |
|                        | In a function, the relationship        |                                           |
|                        | between the domain and range may be    |                                           |
|                        | represented by a rule. This rule may   |                                           |
|                        | be expressed using function notation,  |                                           |
|                        | f(x), which means the value of the     |                                           |
|                        | function at x and is read “f of x”.    |                                           |
|                        | **Zeros of the function** (roots or    | Zeros of the function (roots or solutions) are the x-intercepts of the function and are found algebraically by substituting 0 for y and solving the subsequent equation. An object x in the domain of f is an x-intercept or a zero of a function f if and only if f(x) = 0. |
|                        | solutions) are the x-intercepts of the |                                           |
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|                        | subsequent equation. An object x in    |                                           |
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|                        | zero of a function f if and only if f(x) = 0. |                                           |
## Topic
Algebra and Functions

### Virginia SOL AFDA. 5
The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques.

### Essential Knowledge and Skills

#### Key Vocabulary
- feasibility region
- linear programming
- system of equations
- system of inequalities

The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:
- Model practical problems with systems of linear inequalities.
- Solve systems of linear inequalities with pencil and paper and using a graphing calculator.
- Solve systems of equations algebraically and graphically.
- Identify the feasibility region of a system of linear inequalities.
- Identify the coordinates of the corner points of a feasibility region.
- Find the maximum or minimum value for the function defined over the feasibility region.
- Describe the meaning of the maximum or minimum value within its context

### Essential Questions and Understandings

#### Essential Questions
- What do the various regions mean, including the feasibility region, with regard to the real-world situation being illustrated graphically?
- How does the objective function relate to the real-world situation?
- How can the solution to a system of linear equations be interpreted in terms of the problem’s context?

#### Essential Understandings
- Linear programming models an optimization process. A linear programming model consists of a system of constraints and an objective quantity that can be maximized or minimized.
- Any maximum or minimum value will occur at a corner point of a feasible region.

#### Teacher Notes and Elaborations
A system of equations (simultaneous equations) is two or more equations in two or more variables considered together or simultaneously. The equations in the system may or may not have a common solution. A linear system may be solved algebraically by the substitution or elimination methods, or by graphing. Graphing calculators are used to solve, compare, and confirm solutions.

A system of linear equations with exactly one solution is characterized by the graphs of two lines whose intersection is a single point, and the coordinates of this point satisfy both equations. A point shared by two intersecting graphs and the ordered pair that satisfies the equations characterizes a system of equations with only one solution. A system of two linear equations with no solution is characterized by the graphs of two lines that do not intersect, they are parallel. A system of two linear equations that has infinite solutions is characterized by two graphs that coincide (the graphs will appear to be the graph of one line), and all the coordinates on this one line satisfy both equations.

Systems of two linear equations can be used to represent two conditions that must be satisfied simultaneously.

An inequality is a statement that one quantity is less than (or greater than) another. The nature of the inequality is not changed when adding or subtracting any real number or when multiplying or dividing by a positive real number. However, when multiplying or dividing by negative numbers you must reverse the inequality symbol to maintain a true statement.

Systems of inequalities (simultaneous inequalities) are two or more inequalities in two or more variables that are considered together or simultaneously. The system may or may not have common solutions. Properties of inequality and order can be used to solve inequalities. Practical problems can be interpreted, represented, and solved using linear inequalities.

Set builder notation may be used to represent solution sets of inequalities.

(continued)
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<thead>
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<tr>
<td><strong>Topic</strong></td>
<td><strong>Teacher Notes and Elaborations (continued)</strong></td>
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<tr>
<td>Algebra and Functions</td>
<td>Practical problems, such as those found in science, business and engineering, often lead to systems of inequalities in many unknowns. To solve such real-life problems, find the solution(s) to a system of inequalities.</td>
</tr>
<tr>
<td><strong>Virginia SOL AFDA. 5</strong></td>
<td>Further exploration and practical application of systems of inequalities is developed in linear programming, where the solution is represented in graphic form. Linear programming models an optimization process. It consists of a system of constraints and an objective quantity that can be maximized or minimized.</td>
</tr>
<tr>
<td></td>
<td><em>Linear programming</em> is a technique used to find the maximum or minimum value of an objective function. Linear inequalities are constraints on the variables of the objective function. The solutions to the system of constraints are contained in the feasibility region. The maximum or minimum value of the objective function occurs at a vertex (corner point) of the feasibility region.</td>
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<td>With the use of graphing calculators or computer programs, students can visualize given inequalities, determine the region of feasible solutions, identify appropriate vertices (coordinates of the corner points of a feasibility region), identify the maximum and minimum value of the function defined over the feasibility region, and thereby facilitate problem solving.</td>
</tr>
</tbody>
</table>
### Essential Knowledge and Skills

**Key Vocabulary**

- combination
- complement
- conditional probability
- dependent events
- factorial
- Fundamental Counting Principle
- independent events
- mutually exclusive events
- permutation
- sample space
- theoretical probability

**The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:**

- Compare and contrast permutations and combinations.
- Calculate the number of permutations of \( n \) objects taken \( r \) at a time.
- Calculate the number of combinations of \( n \) objects taken \( r \) at a time.
- Define and give contextual examples of complementary, dependent, independent, and mutually exclusive events.
- Given two or more events in a problem setting, determine if the events are complementary, dependent, independent, and/or mutually exclusive.
- Find conditional probabilities for dependent, independent, and mutually exclusive events.
- Represent and calculate probabilities using Venn diagrams and probability trees.
- Analyze, interpret, and make predictions based on theoretical probability within real-world context.
- Given a real-world situation, determine when to use permutations or combinations.

### Essential Questions and Understandings

**Teacher Notes and Elaborations**

**Essential Questions**

- How are theoretical and experimental probabilities determined?
- How can a Venn diagram be used to illustrate the relationship between events?
- What are dependent, independent, and mutual exclusive events?
- What is the difference between a permutation and a combination?

**Essential Understandings**

- The **Fundamental Counting Principle** states that if one decision can be made \( n \) ways and another can be made \( m \) ways, then the two decisions can be made \( nm \) ways.
- Permutations are used to calculate the number of possible arrangements of objects.
- Combinations are used to calculate the number of possible selections of objects without regard to the order selected.
- A **sample space** is the set of all possible outcomes of a random experiment.
- An event is a subset of the sample space.
- \( P(E) \) is a way to represent the probability that the event \( E \) occurs.
- **Mutually exclusive events** are events that cannot both occur simultaneously.
- If \( A \) and \( B \) are mutually exclusive then \( P(A \cup B) = P(A) + P(B) \).
- The **complement** of event \( A \) consists of all outcomes in which event \( A \) does not occur.
- \( P(B|A) \) is the probability that \( B \) will occur given that \( A \) has already occurred. \( P(B|A) \) is called the **conditional probability** of \( B \) given \( A \).
- Venn diagrams may be used to examine conditional probabilities.

\[
P(B|A) = \frac{P(A \cap B)}{P(A)}
\]

\[
\Rightarrow P(A \cap B) = P(A)P(B|A)
\]
<table>
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<td><strong>Data Analysis</strong></td>
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<tr>
<td><strong>Virginia SOL AFDA. 6</strong></td>
<td>The student will calculate probabilities. Key concepts include: a. conditional probability; b. dependent and independent events; c. addition and multiplication rules; d. counting techniques (permutations and combinations); and e. Law of Large Numbers.</td>
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<td><strong>- Two events, A and B, are independent if the occurrence of one does not affect the probability of the occurrence of the other. If A and B are not independent, then they are said to be dependent.</strong></td>
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<td><strong>- If A and B are independent events, then ( P(A \cap B) = P(A)P(B) ).</strong></td>
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<td>The experimental probability of an event is determined by carrying out a simulation or an experiment. The experimental probability is found by repeating an experiment many times and using the ratio.</td>
<td><strong>Experimental probability</strong> is <strong>( \frac{\text{number of times desired outcomes occur}}{\text{total number of trials in the experiment}} )</strong></td>
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<td>Events are independent when the outcome of one has no effect on the outcome of the other. For example, rolling a number cube and flipping a coin are independent events. The probability of two independent events is found by using the following formula: ( P(A \text{ and } B) = P(A) \cdot P(B) )</td>
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<td>Ex: If a bag holds a blue ball, a red ball, and a yellow ball, what is the probability of picking a blue ball out of the bag on the first pick and then without replacing the blue ball in the bag, picking a red ball on the second pick?</td>
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<td>[ P(\text{blue and red}) = P(\text{blue}) \cdot P(\text{red after blue}) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6} ]</td>
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<td>A factorial is the product of all the positive integers through the given integer (e.g., ( 5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 )).</td>
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<td>A permutation is an arrangement of items in a particular order. It can often be found by using the Fundamental Counting Principle or factorial notation.</td>
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<td>The number of permutation of ( n ) objects is given as ( n! ). The permutations of ( n ) objects taken ( r ) at a time is ( P(n, r) ) or ( _nP_r = \frac{n!}{(n-r)!} ) for ( 0 \leq r \leq n ).</td>
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<td>A combination is a selection in which order does not matter. The number of combinations of ( n ) items taken ( r ) at a time is ( C(n, r) ) or ( _rC_n = \frac{n!}{r!(n-r)!} ) for ( 0 \leq r \leq n ). The formula for combinations is like the formula for permutations except that it contains the factor ( r! ) to compensate for duplicate combinations.</td>
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| **Virginia SOL AFDA. 6** | The student will calculate probabilities. Key concepts include:  
  a. conditional probability;  
  b. dependent and independent events;  
  c. addition and multiplication rules;  
  d. counting techniques (permutations and combinations); and  
  e. Law of Large Numbers. |

**Combination rule:**

The number of combinations of \( r \) items selected from \( n \) different items is: 
\[
\binom{n}{r} = \frac{n!}{(n-r)!r!}
\]

The following conditions must apply: a total of \( n \) different items available, select \( r \) of the \( n \) items without replacement (consider rearrangements of the same items to be the same – the combination of ABC is the same as CBA).

**Example:** A Board of Trustees at the XYZ Company has 9 members. Each year, they elect a 3-person committee to oversee buildings and grounds. Each year, they also elect a chairperson, vice chairperson, and secretary. When the board elects the buildings and grounds committee, how many different 3-person committees are possible?

**Solution:** Because order is irrelevant when electing the buildings and grounds committee, the number of combinations of \( r = 3 \) people selected from the \( n = 9 \) available people; is 
\[
\binom{9}{3} = \frac{9!}{(9-3)!3!} = 84.
\]

Because order does matter with the slates of candidates, the number of sequences (or permutations) of \( r = 3 \) people selected from the \( n = 9 \) available people is 
\[
\frac{9!}{(9-3)!} = \frac{9!}{6!} = 504.
\]

There are 84 different possible committees of 3 board members, but there are 504 different possible slates of candidates.

**Permutation Rule (When Some Items Are Identical to Others)**

If there are \( n \) items with \( n_1 \) alike, \( n_2 \) alike,...\( n_k \) alike, the number of permutations of all \( n \) items is 
\[
\frac{n!}{n_1!n_2!...n_k!}.
\]

**Example:** Consider the letters BBBBBAAAA, which represent a sequence of recent years in which the Dow Jones Industrial Average was below (B) the mean or above (A) the mean. How many ways can the letters BBBBBAAAA be arranged? Does it appear that the sequence is random? Is there a pattern suggesting that it would be wise to invest in stocks?

**Solution:** In the sequence BBBBBAAAA, \( n = 9 \) items, with \( n_1 = 5 \) alike and \( n_2 = 4 \) others that are alike. The number of permutations is computed as follows: 
\[
\frac{9!}{5!4!} = 126.
\]

There are 126 different ways that the letters can be arranged. Because there are 126 different possible arrangements and only two of them (BBBBBAAAA and AAAAABBBBB) result in the letters all grouped together, it appears that the sequence is not random. Because all of the below values occur at the beginning and all of the above values occur at the end, it appears that there is a pattern of increasing stock values. This suggests that it would be wise to invest in stocks.

A permutation problem occurs when different orderings of the same items **are** counted separately. A combination problem occurs when different orderings of the same items **are not** counted separately.
### Curriculum Information

<table>
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<th>Topic</th>
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<tr>
<td>Data Analysis</td>
<td>continuous probability distribution \ deviation \ mean \ median \ mode \ normal curve \ normal distribution \ outlier \ quartile \ range \ standard deviation \ variance \ z-score</td>
</tr>
</tbody>
</table>

### Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:

- Interpret mean, median, mode, range, interquartile range, variance, and standard deviation of a univariate data set in terms of the problem’s context.
- Explain the influence of outliers on a univariate data set.
- Explain ways in which standard deviation addresses dispersion by examining the formula for standard deviation.
- Identify the properties of a normal probability distribution.
- Describe how the standard deviation and the mean affect the graph of the normal distribution.
- Determine the probability of a given event, using the normal distribution.

### Essential Questions and Understandings

- What are outliers and how do they influence data sets?
- How are measures of central tendency used?
- What is meant by the spread of the data?
- What is the variance and standard deviation for a set of data?
- What is the z-score?
- What is a normal distribution curve and how is the graph constructed?
- How can the amount of data that lies within 2, 3, or k standard deviation of the mean be found?
- How does the standard normal distribution curve correspond to probability?
- How can the area under the standard normal curve be found?

### Essential Understandings

- Analysis of the descriptive statistical information generated by a univariate data set includes the relationships between central tendency, dispersion, and position.
- The normal distribution curve is a family of symmetrical curves defined by the mean and the standard deviation.
- Areas under the curve represent probabilities associated with continuous distributions.
- The normal curve is a probability distribution and the total area under the curve is 1.
- The mean of the data in a standard normal density function is 0 and the standard deviation is 1. This allows for the comparison of unlike data.
- The amount of data that falls within 1, 2, or 3 standard deviations of the mean is constant and the basis of z-score data normalization.

### Teacher Notes and Elaborations

Descriptive and visual forms of numerical data help interpret and analyze data from real-world situations.

A univariate data set consists of observations on a single variable.

Box-and-whisker plots can be used to summarize and analyze data. These plots graphically display the median, quartiles, interquartile range, and extreme values (minimum and maximum) in a set of data. They can be drawn vertically or horizontally. A box-and-whisker plot consists of a rectangular box with the ends located at the first and third quartiles. The segments extending from the ends of the box to the extreme values are called whiskers.

The mean of a data set is the sum of the data entries divided by the number of entries.

The mode of a data set is the data entry that occurs with the greatest frequency. If no data entry is repeated, the data set has no mode. If two entries occur with the same greatest frequency, each entry is a mode and the data set is called bimodal.

The range of a data set is the difference between the maximum and minimum data entries in the set.

\[ \text{Range} = (\text{Maximum data entry}) - (\text{Minimum data entry}) \]
### Topic: Data Analysis

**Virginia SOL: AFDA.7**
The student will analyze the normal distribution. Key concepts include:
- characteristics of normally distributed data;
- percentiles;
- normalizing data using z-scores; and
- area under the standard normal curve and probability.

### Teacher Notes and Elaborations (continued)

The median of an odd collection of numbers, arranged in order, is the middle number. The median of an even collection of numbers, arranged in order, is the average of the two middle numbers. The median of an ordered collection of numbers roughly partitions the collection into two halves, those below the median and those above. The first quartile is the median of the lower half. The second quartile is the median of the entire collection. The third quartile is the median of the upper half. Quartiles partition an ordered collection into four quarters. The “whiskers” extend from the minimum to Q1 and Q3 to the maximum. The “box” extends from Q1 to Q3.

<table>
<thead>
<tr>
<th>Minimum Value</th>
<th>Median of Lower Half</th>
<th>Median</th>
<th>Median of Upper Half</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td></td>
<td></td>
<td>Q2</td>
<td></td>
</tr>
<tr>
<td>Q3</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

**Interquartile Range (IQR)** = Q3 − Q1

**Range** = maximum value − minimum value

A box-and-whisker plot makes it easy to see where the data are spread out and where they are concentrated. Multiple box-and-whisker plots can be used to compare and contrast sets of data.

An **outlier** is an item of data with a value substantially different from the rest of the items in the data set. Sometimes an outlier is an important part of the data. At other times, it can represent a false reading.

In formal statistics, the arithmetic **mean** (average) of a population is represented by the Greek letter µ (mu), while the calculated arithmetic mean of a sample is represented by \( \bar{x} \), read “x bar.” In general, a bar over any symbol or variable name in statistics denotes finding its mean.

Deviation of a data entry in a population data set is the difference between the entry and the mean \( \mu \) of the data set. It is the difference from the mean \( x - \bar{x} \), or other measure of center.

Deviation of \( x = x - \mu \)

**Dispersion** is the degree to which the values of a frequency distribution are scattered around some central point, usually the arithmetic mean or median.

**Variance**

One way to address the dilemma of the sum of the deviations of elements from the mean being equal to zero is to square the deviations prior to finding the arithmetic mean. The average of the squared deviations from the mean is known as the **variance**, and is another measure of the spread of the elements in a data set.

\[
\text{Variance} \ (\sigma^2) = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}, \text{ where } \mu \text{ represents the mean of the data set, } n \text{ represents the number of elements in the data set, and } x_i \text{ represents the } i^{th} \text{ element of the data set.}
\]

The differences between the elements and the arithmetic mean are squared so that the differences do not cancel each other out when finding the sum. When squaring the differences, the units of measure are squared and larger differences are “weighted” more heavily than smaller differences. In order to provide a measure of variation in terms of the original units of the data, the square root of the variance is taken, yielding the **standard deviation**.

(continued)
Essential Questions and Understandings

Standard Deviation

The standard deviation is a measure of how each value in a data set varies, or deviates, from the mean. The standard deviation is the positive square root of the variance of the data set. The greater the value of the standard deviation, the more spread out the data are about the mean. The lesser (closer to 0) the value of the standard deviation, the closer the data are clustered about the mean.

\[
\text{Standard deviation (σ)} = \sqrt{\frac{\sum (x_i - \mu)^2}{n}}, \text{ where } \mu \text{ represents the mean of the data set, } n \text{ represents the number of elements in the data set, and } x_i \text{ represents the } i^{th} \text{ element of the data set.}
\]

Often, textbooks will use two distinct formulas for standard deviation. In these formulas, the Greek letter “σ”, written and read “sigma”, represents the standard deviation of a population, and “s” represents the sample standard deviation. The population standard deviation can be estimated by calculating the sample standard deviation. The formulas for sample and population standard deviation look very similar except that in the sample standard deviation formula, \(n - 1\) is used instead of \(n\) in the denominator. The reason for this is to account for the possibility of greater variability of data in the population than what is seen in the sample. When \(n - 1\) is used in the denominator, the result is a larger number. Therefore, the calculated value of the sample standard deviation will be larger than the population standard deviation. As sample sizes get larger (\(n\) gets larger), the difference between the sample standard deviation and the population standard deviation gets smaller. The use of \(n - 1\) to calculate the sample standard deviation is known as Bessel’s correction. Use the formula for standard deviation with \(n\) in the denominator as noted above.

When using Casio or Texas Instruments (TI) graphing calculators to compute the standard deviation for a data set, two computations for the standard deviation are given, one for a population (using \(n\) in the denominator) and one for a sample (using \(n - 1\) in the denominator). Students should be asked to use the computation of standard deviation for population data in instruction and assessments. On a Casio calculator, it is indicated with “\(x\sigma n\)” and on a TI graphing calculator as “\(\sigma x\)”.

z-Scores

A z-score, also called a standard score, is a measure of position derived from the mean and standard deviation of the data set. The z-score will be used to determine how many standard deviations an element is above or below the mean of the data set. It can also be used to determine the value of the element, given the z-score of an unknown element and the mean and standard deviation of a data set. The z-score has a positive value if the element lies above the mean and a negative value if the element lies below the mean. A z-score associated with an element of a data set is calculated by subtracting the mean of the data set from the element and dividing the result by the standard deviation of the data set.

\[
z\text{-score (} z \text{)} = \frac{x - \mu}{\sigma}, \text{ where } x \text{ represents an element of the data set, } \mu \text{ represents the mean of the data set, and } \sigma \text{ represents the standard deviation of the data set.}
\]

A z-score can be computed for any element of a data set; however, they are most useful in the analysis of data sets that are normally distributed.

A normal distribution is a set of continuous random variables where the mean, the median, and the mode are all equal and the graph is symmetric and bell-shaped.
**Essential Questions and Understandings**

### Summary of the Properties of the Normal Distribution

- The curve is bell shaped.
- The mean, median, and mode are equal and located at the center.
- The curve is unimodal, it has only one mode.
- The curve is symmetric about the mean.
- The curve is continuous.
- The curve never touches the x-axis, which is the asymptote to the curve.
- The total area under the curve is equal to 1.00 or 100%.
- The area which lies within one standard deviation is 68%, two standard deviations is 95%, and three standard deviations is 99.7%.

For a distribution that is symmetrical and bell-shaped, the Empirical Rule is used. Approximately 68% of the data values will lie within one standard deviation on each side of the mean. Approximately 95% of the data values will lie within two standard deviations on each side of the mean. Approximately 97% of the data values will lie within three standard deviations on each side of the mean.

A continuous random variable has an infinite number of possible values that can be represented by an interval on the number line. Its probability distribution is called a **continuous probability distribution**. A normal distribution is a continuous probability distribution for a random variable $x$. The graph of a normal distribution is called the **normal curve** and has the following properties:

1. The mean, median, and mode are equal.
2. The normal curve is bell shaped and is symmetric about the mean.
3. The total area under the normal curve is equal to one.
4. The normal curve approaches but never touches the x-axis as it extends farther and farther away from the mean.
5. Between $\mu - \sigma$ and $\mu + \sigma$ (in the center of the curve) the graph curves downward. The graph curves upward to the left of $\mu - \sigma$ and to the right of $\mu + \sigma$. The points at which the curve changes from curving upward to curving downward are called inflection points.

![Inflection points](image-url)

More information (keystrokes and screenshots) on using graphing calculators to compute this is found in the Algebra I curriculum guide under Virginia SOL 9 in the Sample Instructional Strategies and Activities.
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<tr>
<td><strong>Virginia SOL AFDA.8</strong></td>
<td>The student will use problem solving, mathematical communication, mathematical reasoning, connections and representations to:</td>
<td><strong>Essential Questions</strong></td>
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<tr>
<td></td>
<td>a. Compare and contrast controlled experiments and observational studies and the conclusions one may draw from each.</td>
<td>• How are experiments/surveys designed?</td>
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<td>b. Identify biased sampling methods.</td>
<td>• What are sampling techniques and how do they reduce bias?</td>
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<td></td>
<td>c. Select a data collection method appropriate for a given context.</td>
<td>• What are the various methods of data collection?</td>
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<td></td>
<td>d. Investigate and describe sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling.</td>
<td>• How does data collection affect conclusions for a problem?</td>
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<td></td>
<td>e. Determine which sampling technique is best, given a particular context.</td>
<td>• What are the differences between controlled experiments and observational studies?</td>
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<td>f. Plan and conduct an experiment or survey. The experimental design should address control, randomization, and minimization of experimental error.</td>
<td>• What do the results of experiments and surveys mean and what are the implications to real-life situations?</td>
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<td>g. Design a survey instrument.</td>
<td>• What analyses and interpretations of data can be obtained through experiments and surveys?</td>
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<tr>
<td><strong>Key Vocabulary</strong></td>
<td>biased</td>
<td><strong>Essential Understandings</strong></td>
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<tr>
<td></td>
<td>biased sample</td>
<td>• The value of a sample statistic may vary from sample to sample, even if the simple random samples are taken repeatedly from the population of interest.</td>
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<td></td>
<td>biased sampling</td>
<td>• Poor data collection can lead to misleading and meaningless conclusions.</td>
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<td></td>
<td>census</td>
<td>• The purpose of sampling is to provide sufficient information so that population characteristics may be inferred.</td>
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<td></td>
<td>cluster</td>
<td>• Inherent bias diminishes as sample size increases.</td>
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<td></td>
<td>cluster sample</td>
<td>• Experiments must be carefully designed in order to detect a cause-and-effect relationship between variables.</td>
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<td></td>
<td>control group</td>
<td>• Principles of experimental design include comparison with a control group, randomization, and blindness.</td>
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<td>observational study</td>
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<td>population</td>
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<td>sample survey</td>
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<td>simple random sample</td>
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<td>stratified sample</td>
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<td>survey</td>
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### Virginia SOL AFDA.8
The student will design and conduct an experiment/survey. Key concepts include:
- sample size;
- sampling technique;
- controlling sources of bias and experimental error;
- data collection; and
- data analysis and reporting.

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**Statistical Studies**

- **Observational** (Observe and measure but do not modify)
- **Experimental** (Apply some treatment)

- **Differences**
- **Impose some treatment on individuals in order to observe their responses. The purpose of an experiment is to study whether the treatment causes a change in the response.**

The design of a statistical study is *biased* if it systematically favors certain outcomes. A sampling method is biased if it tends to give samples in which some characteristic of the population is underrepresented or overrepresented (*biased sampling*).

A *biased sample* has a distribution that is not determined only by the population from which it is drawn, but also by some property that influences the distribution of the sample. Biased samples do not represent the entire population of the study. For example, an opinion poll might be biased by geographical location.

Another source of bias is voluntary response samples. These are biased because people with strong opinions are more likely to respond.

A *cluster* is a naturally occurring subgroup of a population used in stratified sampling. A *cluster sample* is when a population falls into a naturally occurring subgroup, which has similar characteristics.

*Simple random sampling* is the process of collecting samples devised to avoid any interference from any shared property of, or relation between the elements selected, so that its distribution is affected only by that of the whole population and can therefore be taken to be representative of it.

A *stratified sample* is a sample that is not drawn at random from the whole population, but is drawn separately from a number of disjoint strata of the population in order to ensure a more representative sample. To achieve a stratified random sample, divide the units of the sampling frame into non-overlapping subgroups and choose a simple random sample from each subgroup.

A type of sample that often leads to biased studies is a convenience sample. A *convenience sample* consists only of available members of the population or members of the population that are easiest to reach. For this reason, this is not a recommended sampling technique.

Sample selection bias (convenience sampling) is the extent to which a sampling procedure produces samples that tend to result in numerical summaries that are systematically too high or too low.